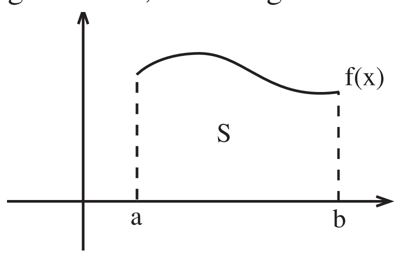
**f, [a, b] de sürekli bir fonksiyon olsun f nin eğrisi x=a, x=b doğruları ve x-ekseni ile alan ; $S = \int_a^b |f(x)|\, dx$ dir.**

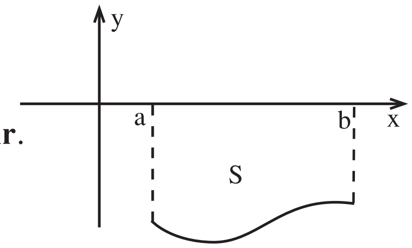



**Alan, x- ekseninin üstünde ise**

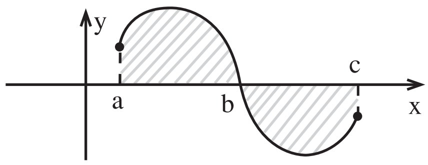
**$\forall x \in [a, b]$ için $f(x) \geq 0 \Rightarrow S = \int_a^b f(x)\, dx$ dir.**

**Alan, x- ekseninin altında ise**

**$\forall x \in [a, b]$ için $f(x) \leq 0 \Rightarrow S = -\int_a^b f(x)\, dx$ dir.**




**Alan, x ekseninin hem altında hem de üstünde ise f, [a,c] de sürekli, $\forall x \in [a, c]$ alan $\int_a^b f(x)\, dx - \int_b^c f(x)\, dx$ dir.**



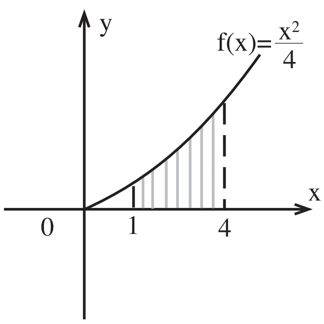

**Örnekler :**

f(x)= 2x doğrusu x-ekseni x=1 ve x= 2 doğrularıyla sınırlanan bölgenin alanını bulunuz.

**1.** $\int_1^2 2x\,dx = x^2 \Big|_1^2 = 2^2 - 1^2 = 4-1 = 3$ bulunur.

**2.** $f(x) = \frac{x^2}{4}$ eğrisi, x-ekseni, x=1 ve x=4 doğrularıyla sınırlanan alanı bulunuz.

**Çözüm :**




$$S = \int_1^4 f(x)\, dx = \int_1^4 \frac{x^2}{4}\, dx$$

$$= \frac{1}{4}\frac{x^3}{3}\Big|_1^4 = \frac{1}{12}x^3 \Big|_1^4$$

$$= \frac{1}{12} \cdot 4^3 - \frac{1}{12} \cdot 1^3 = \frac{64}{12} - \frac{1}{12} = \frac{63}{12}\ br^2$$

Görüldüğü üzere, integral alma sayesinde parçalama yönteminden daha basit bir yöntemle alanı hesapladık.

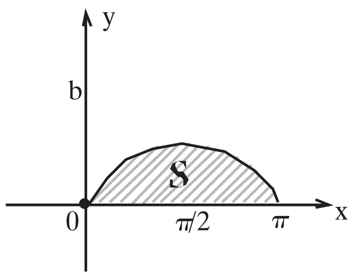
f(x) = Sinx eğrisinin [0, π] aralığında kalan parçası ve x- ekseni ile sınırlanan alanı hesaplayınız.

$$S = \int_0^{\pi} \text{Sinx}\, dx = -\text{Cosx}\Big|_0^{\pi}$$

$$= -(\text{Cos}\pi - \text{Cos}0)$$

$$= -(-1) + 1 = 1+1$$

$$= 2br^2$$

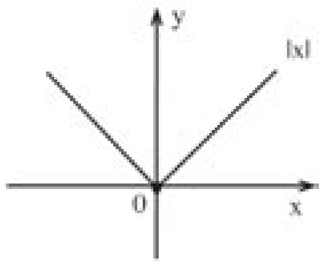



$\int_{-2}^{3} |x|\, dx$ integralini hesaplayınız.

$$|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases} \Rightarrow \int_a^b f(x)dx - \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$$

C∈[a, b] olduğuna göre

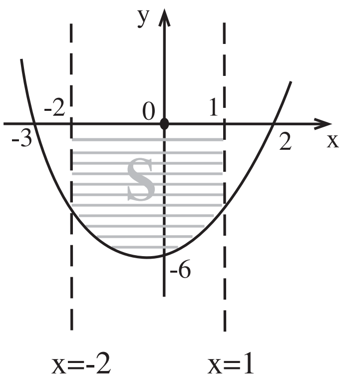
$$\int_{-2}^{3} |x|\, dx = \int_{-2}^{0} -x\, dx + \int_{0}^{3} x\, dx = -\frac{x^2}{2}\Big|_{-2}^{0} + \frac{x^2}{2}\Big|_{0}^{3}$$




$$= -\left[0^2 - \frac{(-2)^2}{2}\right] + \left(\frac{9}{2} - 0\right) = \frac{4}{2} + \frac{9}{2} = \frac{13}{2}$$ bulunur.

**5.** $f : R \rightarrow R$ ; $f(x) = x^2+x-6$ eğrisi, $x = -2$, $x = 1$ doğruları ve x- ekseni ile sınırlanan bölgenin alanını bulunuz.

**Çözüm :**




$$S = -\int_{-2}^{1} f(x)\,dx = -\int_{-2}^{1} (x^2+x-6)\,dx$$

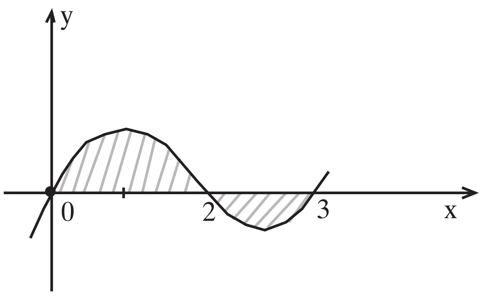
$$= -\left(\frac{x^3}{3} + \frac{x^2}{2} - \frac{6x}{1}\right)\Bigg|_{-2}^{1} = \left(-\frac{1}{3} - \frac{1}{2} + 6\right) +$$

$$\left(\frac{-8}{3} + \frac{4}{2} + 12\right) = \frac{-2 -3 +36 - 16 + 12 +72}{6} = \frac{99}{6}$$

$$= \frac{33}{2}\ br^2$$

**6.** $f(x) = x^3 - 5x^2 +6x$ fonksiyonunun eğrisi ile x- ekseninin sınırladığı bölgenin alanını bulunuz.

**Çözüm :** $x(x^2-5x+6) = x(x-3)\ (x-2) = 0$ ise $x=0, x=3, x=2$




$$S = \int_{0}^{2} (x^3-5x^2+6x)\,dx - \int_{2}^{3} (x^3-5x^2+6x)\,dx = \left(\frac{x^4}{4} - \frac{5}{3}x^3+3x^2\right)\Bigg|_{0}^{2}$$

$$-\left(\frac{x^4}{4} - \frac{5}{3}x^3 +3x^2\right)\Bigg|_{2}^{3} = \left(4 - \frac{40}{3} +12\right) - \left(\frac{81}{4} - 45+27-4 + \frac{40}{3} - 12\right)$$

$$= 4 - \frac{40}{3} + 12 - \frac{81}{4} + 45 - 27 + 4 - \frac{40}{3} + 12 = \underset{(12)}{\frac{50}{1}} - \underset{(4)}{\frac{80}{3}} - \underset{(3)}{\frac{81}{4}} = \frac{600-563}{12} = \frac{37}{12}$$

**7.** $\int_0^2 |x-1|\,dx$ integralini hesaplayınız.

**Çözüm :** x =1 kritik nokta

$$|x-1| = \begin{cases} x-1 \geq 0 \text{ ise } & x-1 \\ x-1<0 \text{ ise } & -(x-1) \end{cases} = \begin{cases} x \geq 1 & ; \quad x-1 \\ x \leq 1 & ; \quad -(x-1) \text{ olduğundan} \end{cases}$$

$$\int_0^2 |x-1|\,dx = \int_0^1 -(x-1)\,dx + \int_1^2 (x-1)\,dx = \left(-\frac{x^2}{2}+x\right)\Big|_0^1 + \left(\frac{x^2}{2}-x\right)\Big|_1^2$$

$$= \frac{-1}{2} + 1 + 2 - 2 - \frac{1}{2} + 1 = -\frac{2}{2} + 2 = -1 + 2 = 1 \text{ bulunur.}$$

**8.** $\int_0^3 [|x|]\,x\,dx$ integralini hesaplayınız.

**Çözüm :**

$x \in [0, 1) \Rightarrow f(x) = [|x|] = 0$

$x \in [1, 2) \Rightarrow f(x) = [|x|] = 1$

$x \in [2, 3) \Rightarrow f(x) = [|x|] = 2$

$$\int_0^3 [|x|]x\,dx = \int_0^1 0\,dx + \int_1^2 x\,dx + \int_2^3 2x\,dx = 0 + \frac{x^2}{2}\Big|_1^2 + x^2\Big|_2^3$$

$$2 - \frac{1}{2} + 9 - 4 = \frac{3}{2} + 5 = \frac{13}{2} \text{ bulunur.}$$

**9.** $\int_{-1}^4 \text{sgn}\,(x^2 - 3x+2)dx$ integralini hesaplayınız.

**Çözüm :**

$$\text{Sgn } f(x) = \begin{cases} 1 \; ; & f(x) > 0 \text{ ise} \\ 0 \; ; & f(x) = 0 \text{ ise} \\ -1 \; ; & f(x) < 0 \text{ ise} \end{cases}$$

f(x) in işaretini inceleyelim.

| x | $-\infty$ | 1 | | 2 | $+\infty$ |
|---|---|---|---|---|---|
| $x^2 - 3x+2$ | + | 0 | − | 0 | + |

$$\int_{-1}^{4} \text{Sgn}\,(x^2-3x+2)dx = \int_{-1}^{1} dx + \int_{1}^{2} -1dx + \int_{2}^{4} 1dx =$$

$$x\Big|_{-1}^{1} + (-x)\Big|_{1}^{2} + x\Big|_{2}^{4} = (1+1) - (2-1) + (4-2) = 2 - 1+2 = 3 \text{ bulunur.}$$

**10.** $x \neq 0$ için $\int_{-1}^{2} [|x|]^{\text{Sgn}x} dx$ integralini hesaplayınız.

**Çözüm :**

$x \in [-1, 0) \Rightarrow [|x|] = -1$

$x \in [0, 1) \Rightarrow [|x|] = 0$

$x \in [1, 2] \Rightarrow [|x|] = 1$

$$\text{Sgn}x = \begin{cases} 1 \;;\; x>0 \text{ ise} \\ -1 \;;\; x<0 \text{ ise} \\ 0 \;;\; x=0 \text{ ise} \end{cases}$$

$$\int_{-1}^{2} [|x|]^{\text{Sgn}x} dx = \int_{-1}^{0} (-1)^{-1} dx + \int_{0}^{1} 0\,dx + \int_{1}^{2} 1\,dx$$

$$= \int_{-1}^{0} -dx + \int_{1}^{2} dx = -x\Big|_{-1}^{0} + x\Big|_{1}^{2} = -1 + 2 - 1 = 0$$

**11.** $\int_{0}^{\frac{\pi}{2}} |\text{Cos}x - \text{Sin}x|\, dx$ integralini hesaplayınız.

**Çözüm :**

$\left[0, \frac{\pi}{2}\right]$ aralığında Cosx -Sinx = 0 denkleminin kökü $x = \frac{\pi}{4}$ tür.

$x \leq \frac{\pi}{4}$ için $\text{Cos}x \geq \text{Sin}x$ ve $|\text{Cos}x-\text{Sin}x| = \text{Cos}x - \text{Sin}x$

$x > \frac{\pi}{4}$ için $\text{Cos}x < \text{Sin}x$ ve $|\text{Cos}x - \text{Sin}x| = -(\text{Cos}x - \text{Sin}x)$

$= \text{Sin}x - \text{Cos}x$ dir.

$$\int_0^{\frac{\pi}{2}} |\text{Cosx} - \text{Sinx}|\, dx = \int_0^{\frac{\pi}{4}} (\text{Cosx} - \text{Sinx})\, dx + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (\text{Sinx} - \text{Cosx})\, dx$$

$$\int_0^{\frac{\pi}{4}} \text{Cosx}\, dx + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \text{Sinx}\, dx - \int_0^{\frac{\pi}{4}} \text{Sinx}\, dx - \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \text{Cosx}\, dx$$

$$= \text{Sinx}\Big|_0^{\frac{\pi}{4}} - \text{Cosx}\Big|_{\frac{\pi}{4}}^{\frac{\pi}{2}} + \text{Cosx}\Big|_0^{\frac{\pi}{4}} - \text{Sinx}\Big|_{\frac{\pi}{4}}^{\frac{\pi}{2}}$$

$$= (\text{Sin}\frac{\pi}{4} - \text{Sin}0) - \left(\text{Cos}\frac{\pi}{2} - \text{Cos}\frac{\pi}{4}\right) + \left(\text{Cos}\frac{\pi}{4} - \text{Cos}0\right) - \left(\text{Sin}\frac{\pi}{2} - \text{Sin}\frac{\pi}{4}\right)$$

$$= \left(\frac{\sqrt{2}}{2} - 0\right) - \left(0 - \frac{\sqrt{2}}{2}\right) + \left(\frac{\sqrt{2}}{2} - 1\right) - \left(1 - \frac{\sqrt{2}}{2}\right)$$

$$= \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} - 1 - 1 + \frac{\sqrt{2}}{2} = \frac{4}{2}\sqrt{2} = 2\left(\sqrt{2} - 1\right) \text{ bulunur.}$$

**12.** Aşağıdaki integralleri hesaplayalım.

**a)** $$\int_1^3 (x^2 - 4x + 2)\, dx = \left(\frac{x^3}{3} - \frac{4}{2}x^2 + 2x\right)\Big|_1^3 = \frac{27}{3} - 18 + 6 - \frac{1}{3} + 2 - 2 =$$

$$-3 - \frac{1}{3} = -\frac{10}{3} \text{ bulunur.}$$

$(\text{Cos}(-\theta) = \text{Cos}\theta\ ;\ \text{Sin}(-\theta) = -\text{Sin}\theta)$

**b)** $$\int_{-1}^3 (2\text{Sinx} + 2\text{Cosx})\, dx = (-2\,\text{Cosx} + 2\,\text{Sinx})\Big|_{-1}^1 = -2\,(\text{Cos}1 - \text{Cos}(-1))$$

$$+2\,(\text{Sin}(1) - \text{Sin}(-1)) = -2\,(\text{Cos}(1) - \text{Cos}(1)) + 2\,(\text{Sin}(1) + \text{Sin}(1)) = 4\,\text{Sin}1$$

**c)** $$S = \int_0^{\pi/4} \frac{1}{\text{Cos}^2 x}\, dx = \text{tgx}\Big|_0^{\frac{\pi}{4}} = \tan\frac{\pi}{4} - \text{tg}0 = 1 - 0 = 1$$

**d)** $$S = \int_0^{\pi/2} \frac{-1}{\sqrt{1-x^2}}\, dx = \text{Arc cosx}\Big|_0^{\frac{\pi}{2}} = \text{Arc cos}\frac{\pi}{2} - \text{Arc cos}0$$

**e)** $S = \int_0^1 \frac{1}{1+x^2}\,dx = Arctgx \Big|_0^1 = Arctg\,1 - Arctg0 = \frac{\pi}{4} - 0 = \frac{\pi}{4}$

**f)** $S = \int_0^1 e^{5x}\,dx = \frac{1}{5}\,e^{5x}\Big|_0^1 = \frac{1}{5}\,(e^{5.1} - e^{5.0}) = \frac{1}{5}\,(e^5 - 1)$

$= \frac{e^5}{2} - \frac{1}{5}$ bulunur.

**g)** $S = \int_{-\pi}^{\pi} Sin\,|x|\,dx = \int_{-\pi}^{0} Sin(-x)\,dx + \int_0^{\pi} Sinx\,dx = \int_{-\pi}^{0} -Sinx\,dx + \int_0^{\pi} Sinx\,dx =$

$Cosx\Big|_{-\pi}^{0} - Cosx\Big|_0^{\pi} = Cos0 - Cos(-\pi) - Cos\pi + Cos0 = 1 - (-1) - (-1) + 1 =$

$1+1+1+1 = 4$ bulunur.

**h)** $\int_{-1}^{2} Sgn[|x|]\,dx = \int_{-1}^{0} Sgn\,(-1)\,dx + \int_0^1 Sgn\,(0)\,dx + \int_1^2 Sgn(1)\,dx$

$= \int_{-1}^{0} -dx + \int_0^1 0dx + \int_1^2 dx = -x\Big|_{-1}^{0} + x\Big|_1^2 = (-1) + 2 - 1 = 0$ dır.

**ı)** $\int_1^4 [|x|]^{[|x|]}\,dx = \int_1^2 1^1dx + \int_2^3 2^2\,dx + \int_3^4 3^3dx =$

$x\Big|_1^2 + 4x\Big|_2^3 + 27x\Big|_3^4 = 2-1+12-8+108-81 = 86$

**j)** $\int_{\pi}^{2\pi} [|Sinx|]\,Sinx\,dx = \int_{\pi}^{2\pi} -Sinx\,dx = Cosx\Big|_{\pi}^{2\pi} = Cos2\pi - Cos\pi$

$= 1-(-1) = 1+1 = 2$ dir.

$x$, $\pi$ den $2\pi$ ye kadar değiştiğinde Sinx, -1 ile 0 arasında değişir. $[|Sinx|] = -1$ dir.

**k)** $\int_0^3 |x^2-3x+2|\,dx = \int_0^1 (x^2-3x+2)dx + \int_1^2 -(x^2-3x+2)\,dx + \int_2^3 (x^2-3x+2)\,dx$

$= \left(\frac{x^3}{3} - \frac{3}{2}x^2+2x)\right)\Big|_0^1 - \left(\frac{x^3}{3}-\frac{3}{2}x^2+2x\right)\Big|_1^2 + \left(\frac{x^3}{3}-\frac{3}{2}x^2+2x\right)\Big|_2^3$

$= \left(\frac{1}{3} - \frac{3}{2} +1\right) - \left(\frac{8}{3} + \frac{12}{2} +4 - \frac{1}{3} + \frac{3}{2} - 2\right) + \left(\frac{27}{3} - \frac{27}{2} + 6 - \frac{8}{3} + \frac{12}{2} - 4\right)$

$= \left(\frac{2-9+6}{6}\right) - \left(\frac{16+36+24-2+9-12}{6}\right) + \left(\frac{54-81+36-16+36-24}{6}\right) - \frac{1+71+5}{6} = \frac{75}{6} = \frac{25}{2}$

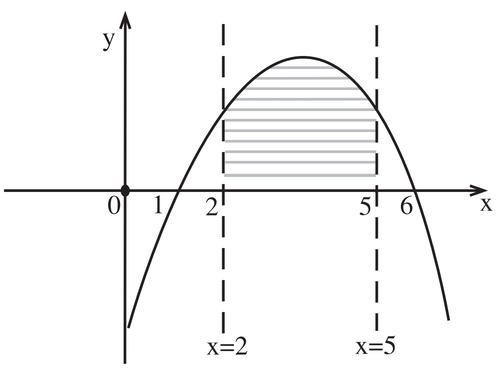
**l)** $\int_1^4 Sgn(x^2-5x+6)\,dx = \int_1^2 1dx - \int_2^3 1dx + \int_3^4 1dx = x\Big|_1^2 - x\Big|_2^3 + x\Big|_3^4 =$

| x | | 2 | | 3 | |
|---|---|---|---|---|---|
| $x^2-5x+6$ | + | 0 | - | 0 | + |

2-1-(3-2) + 4-3 = 1-1+1 = 1 bulunur.

**13.** f: (x) = $-x^2+7x-6$ fonksiyonunun eğrisi x = 2, x=5 doğruları ve x-ekseni ile sınırlanan bölgenin alanını bulunuz.

**Çözüm :**




$S = \int_2^5 f(x)\,dx = \int_2^5 (-x^2+7x-6)\,dx$

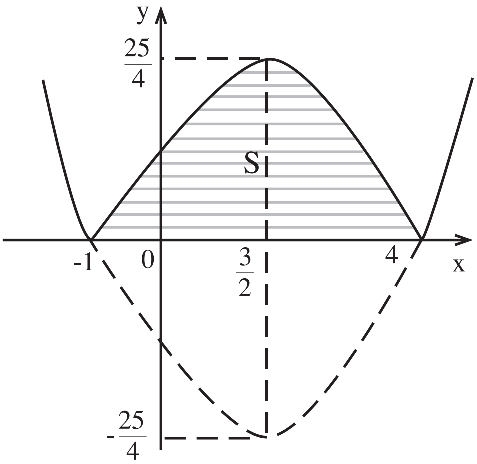
$= \left(-\frac{x^3}{3} + \frac{7x^2}{2} - 6x\right)\Big|_2^5$

$= \left(-\frac{125}{3} + \frac{175}{2} - 30\right) - \left(-\frac{8}{3} + 14-12\right)$

$= \frac{189}{2}$

**14.** $f(x) = |x^2-3x-4|$ fonksiyonunun eğrisi ile x-ekseninin sınırlandığı bölgenin alanını bulunuz.

**Çözüm :**



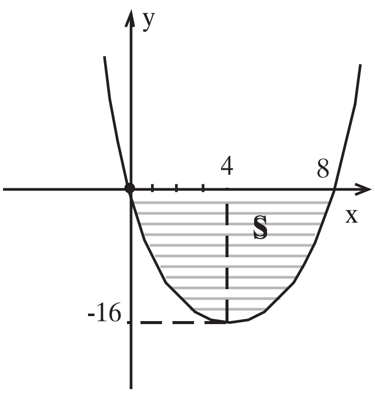

$$f(x) = \left|\left(x-\frac{3}{2}\right)^2 - \frac{25}{4}\right|$$

$$S = -\int_{-1}^{4} (x^2-3x-4)\, dx = \left(-\frac{x^3}{3} + \frac{3}{2}x^2 + 4x\right)\Big|_{-1}^{4}$$

$$= \left(-\frac{64}{3} + 24 + 16\right) - \left(\frac{1}{3} + \frac{3}{2} - 4\right)$$

$$= \frac{141}{6}$$

**15.** $f(x) = x^2 - 8x$ fonksiyonunun eğrisine x -ekseni ile sınırlanan bölgenin alanını bulunuz.




$$f(x) = x^2 - 8x = (x-4)^2 - 16$$

$$S = -\int_{0}^{8} f(x)\, dx = \int_{0}^{8} (x^2-8x)\, dx$$

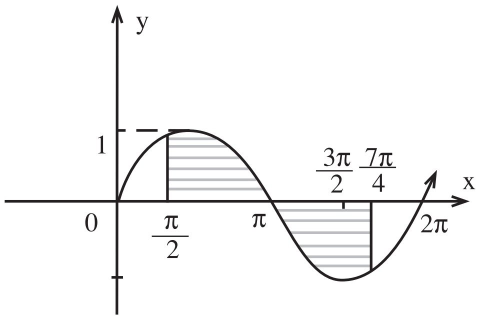
$$= \left(\frac{x^3}{3} - 4x^2\right)\Big|_{0}^{8} =$$

$$-\left(\frac{8^3}{3} - 4.64\right) = \frac{512}{3} - 256$$

$$= \frac{256}{3}\ br^2 \text{ bulunur.}$$

**16.** f(x) =Sinx fonksiyonunun eğrisi ile $x = \frac{\pi}{2}$, $x = \frac{7\pi}{4}$ doğruları ve x- ekseni ile sınırlanan bölgenin alanını bulunuz.

**Çözüm :**




$$S=\int_{\frac{\pi}{2}}^{\pi} \text{Sinx dx}-\int_{\pi}^{\frac{7\pi}{4}} \text{Sinx dx} = -\text{Cosx}\Big]_{\frac{\pi}{2}}^{\pi} + \text{Cosx}\Big]_{\pi}^{\frac{7\pi}{4}}$$

$$= -\text{Cos}\pi + \text{Cos}\frac{\pi}{4} + \text{Cos}\frac{7\pi}{4} - \text{Cos}\pi = 1+\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} +1 = 2+\sqrt{2}\ \text{br}^2$$

**17.** Aşağıdaki integralleri hesaplayınız.

**a)** $$\int_0^5 \text{Sin}\frac{[|x|]\,\pi}{2}\text{dx} = \int_0^1 \text{Sin0 dx} + \int_1^2 \text{Sin}\frac{\pi}{2}\text{ dx} + \int_2^3 \text{Sin}\pi\text{ dx}$$

$$+\int_3^4 \text{Sin}\frac{3\pi}{2}\text{dx} + \int_4^5 \text{Sin }2\pi\text{ dx} =$$

$$= \int_0^1 0\text{ dx} + \int_1^2 1\text{ dx} + \int_2^3 0\text{ dx} + \int_3^4 -\text{dx} + \int_4^5 0\text{ dx} =$$

$$\int_1^2 \text{dx} - \int_3^4 \text{dx} = \Big|_1^2 x - \Big|_3^4 x = (2\text{-}1) - (4\text{-}3) = 1\text{-}1 = 0$$

**b)** $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{-1}{\operatorname{Sin}^2 x} dx = \operatorname{Cotg}x \Big|_{\frac{\pi}{4}}^{\frac{\pi}{2}} = \operatorname{Cotg}\frac{\pi}{2} - \operatorname{Cotg}\frac{\pi}{4} = 0-1= -1$

**c)** $\int_{1}^{e} \frac{1}{x} dx = \ln x \Big|_{1}^{e} = \ln e - \ln 1 = 1-0 = 1$ dir.

## İKİ EĞRİ İLE SINIRLANAN BÖLGENİN ALANI

**Örnekler :**

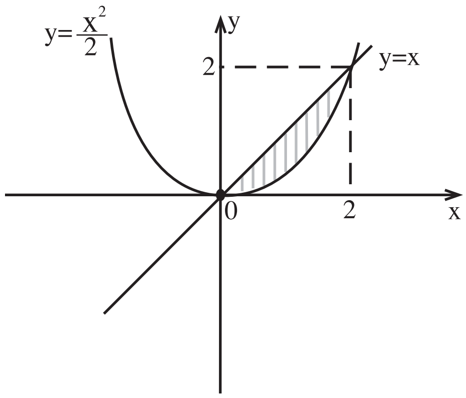
**1.** $y = x$ doğrusu ve $y = \frac{x^2}{2}$ parabolünün sınırladığı bölgenin alanını bulunuz.

**Çözüm :**

Eğriyle doğruyu birlikte çözelim ve sınırlarını bulalım. Sonra grafiğini çizip, arada kalan bölgeyi tanımlayalım.

$x = \frac{x^2}{2} \Rightarrow x^2 - 2x = 0 \Rightarrow x(x-2) = 0 \Rightarrow$

$x_1 = 0$ veya $x-2 = 0 \Rightarrow x_2 = 2$ dir.

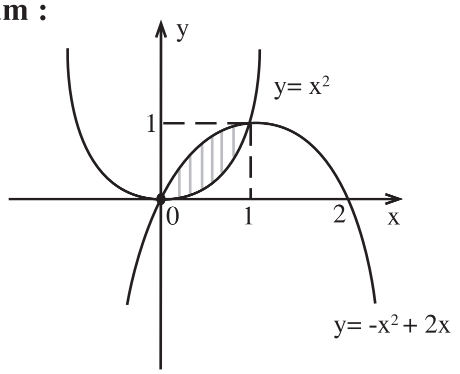



$S = \int_0^2 (x - \frac{x^2}{2})\, dx = \int_0^2 x\, dx - \int_0^2 \frac{x^2}{2} dx$

$= \frac{x^2}{2}\Big|_0^2 - \frac{x^3}{6}\Big|_0^2 = 2 - \frac{4}{3} = \frac{2}{3}\ \text{br}^2$

**2.** $y = x^2$, $y = -x^2+2x$ fonksiyonlarının eğrileri ile sınırlı bölgenin alanını bulunuz.

**Çözüm :**



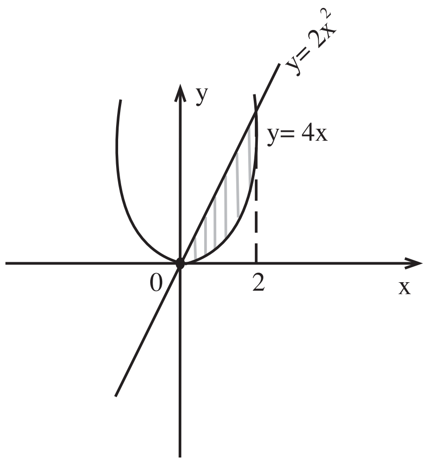

$y = -x^2+2x = -(x-1)^2+1$

$$S = \int_0^1 (-x^2+2x-x^2)\,dx = -2\int_0^1 x^2\,dx + 2\int_0^1 x\,dx = -2\left.\frac{x^3}{3}\right|_0^1 + 2\left.\frac{x^2}{2}\right|_0^1$$

$$= -\frac{2}{3}(1-0) + (1-0) = -\frac{2}{3} + 1 = \frac{1}{3}\ br^2$$

**3.** $y = 2x^2$ eğrisi ve $y = 4x$ doğrusu ile sınırlanan bölgenin alanını bulunuz.

**Çözüm :**




Eğri ile doğruyu ortak çözelim.

$4x = 2x^2 \Rightarrow 2x = x^2$

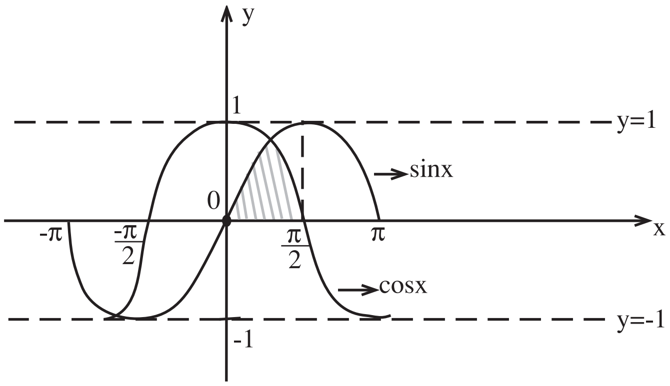
$x_1 = 0$ veya $x_2 = 2$

$$S = \int_0^2 (4x-2x^2)\,dx = 4\int_0^2 x\,dx - 2\int_0^2 x^2\,dx = 4\left.\frac{x^2}{2}\right|_0^2 - 2\left.\frac{x^3}{3}\right|_0^2$$

$$= 2.4 - \frac{16}{3} = 8 - \frac{16}{3} = \frac{8}{3}\ br^2$$

**4.** $\left[0, \frac{\pi}{2}\right]$ aralığında, y = Sinx , y = Cosx eğrileri ve x- ekseni ile sınırlanan bölgenin alanını bulunuz.

**Çözüm :**

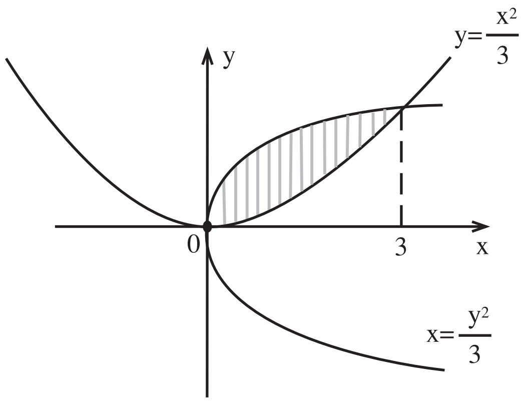



$$S = \int_0^{\pi/4} \text{Sinx}\,dx + \int_{\pi/4}^{\pi/2} \text{Cosx}\,dx = -\text{Cosx}\Big|_0^{\frac{\pi}{4}} + \text{Sinx}\Big|_{\frac{\pi}{4}}^{\frac{\pi}{2}}$$

$$= -\,\text{Cos}\frac{\pi}{4} + \text{Cos}0 + \text{Sin}\frac{\pi}{2} - \text{Sin}\frac{\pi}{4} = -\frac{\sqrt{2}}{2} + 1 + 1 - \frac{\sqrt{2}}{2} = 2 - \sqrt{2}$$

**5.** $y^2 = 3x$ ve $x^2 = 3y$ eğrileri ile sınırlanan bölgenin alanını bulunuz.

**Çözüm :**




$y = \frac{x^2}{3}$

$(\frac{x^2}{3})^2 = 3x$

$\frac{x^4}{9} = 3x$

$x^4 - 27x = 0$

$x(x^3 - 27) = 0$

$x_1 = 0$ veya $x_2 = 3$

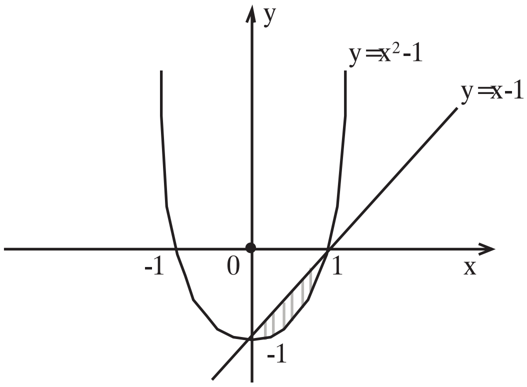
$x = \frac{y^2}{3} \Rightarrow y = \sqrt{3x}$

$$S = \int_0^3 \left(\sqrt{3x} - \frac{x^2}{3}\right) dx = \int_0^3 \sqrt{3}\,\sqrt{x}\,dx - \frac{1}{3}\int_0^3 x^2\,dx =$$

$$= \frac{2\sqrt{3}}{3}\sqrt{x^3}\Big|_0^3 - \frac{x^3}{9}\Big|_0^3 = \frac{2\sqrt{3}}{3}\,2\sqrt{3}\,\sqrt{27} - 3 = 12\sqrt{3} - 3 \text{ br}^2$$

**6.** $y = x^2 - 1$ eğrisi ve $y = x - 1$ doğrusunun sınırlandığı bölgenin alanını bulunuz.

**Çözüm :**

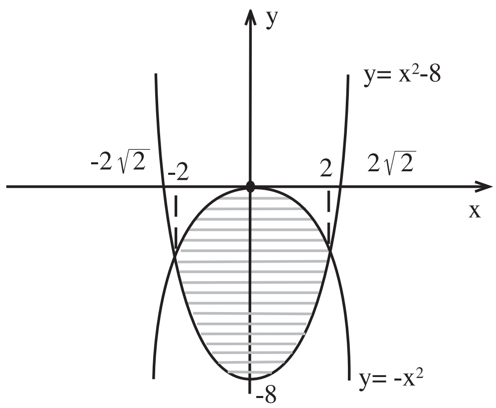



$$S = \int_0^1 \left[(x-1) - (x^2-1)\right] dx = \int_0^1 (x-x^2)\, dx = \left(\frac{x^2}{2} - \frac{x^3}{3}\right)\Big|_0^1 =$$

$$\left(\underset{(3)}{\frac{1}{2}} - \underset{(2)}{\frac{1}{3}}\right) = \frac{3}{6} - \frac{2}{6} = \frac{1}{6}\ br^2$$

**7.** $y = x^2 - 8$ ve $y = -x^2$ eğrileri ile sınırlanan bölgenin alanını bulunuz.

**Çözüm :**




$x^2 - 8 = -x^2$

$2x^2 = 8$

$x^2 = 4$

$x_1 = 2\ ,\ x_2 = -2$

$$S = \int_{-2}^{2} (-x^2 - x^2 + 8)\, dx = \int_{-2}^{2} (-2x^2 + 8)\, dx = \left(\frac{-2}{3} x^3 + 8x\right)\Big|_{-2}^{2}$$

$$= \left(-\frac{2}{3}\,8 + 8.2\right) - \left(\frac{-2}{3}\,(-8) - 16\right) = -\frac{16}{3} + 16 - \frac{16}{3} + 16$$

$$= -\underset{(1)}{\frac{32}{3}} + \underset{(3)}{\frac{32}{1}} = \frac{96-32}{3} = \frac{64}{3}\ br^2$$